

Efficient Frontier Computation In the Markowitz Model

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Introduction

Gaining money without taking too much risk is ultimately the way people would prefer to invest, as being risk seeking is quite rare. An investor’s goal is to create as much return as possible whilst keeping risk to a minimum. One way of doing this is by using the Markowitz model, founded by Henry Markowitz in 1952, to create an optimisation problem that calculates a percentage of how much of the investment should be put into each asset. This percentage is called the weight of the invested money. Calculating these values for a series of companies and plotting them on the efficient frontier is the main objective of this report. Markowitz found that the efficient frontier was a way of showing the optimal portfolios calculated. The most optimal point, where risk is at a minimum whilst giving maximum return, is usually somewhere to the far left of the efficient frontier (Markowitz, 1952).

Data Collection

In order to calculate the model, the historical prices of different companies needed to be gathered. It would be expected that using companies from different industry sectors would show a higher correlation, rather than using companies at random. It would be interesting to look at how the companies moved with each other in the final results. So, choosing 20 companies, from four sectors and calculating the average close and standard deviation made way for the production of the correlation and covariance matrices. The covariance matrix was then inputted into the Markowitz Model that calculates the optimum portfolios.

The Markowitz Model

Using the quadratic program minimise

$$\frac{1}{2} \sum_{i,j=1}^n w_i w_j \sigma_{ij}$$

subject to

$$\sum_{i=1}^n w_i \bar{r}_i = \bar{r}$$
$$\sum_{i=1}^n w_i = 1$$

(Luenberger, 1995)
with a fixed value of \bar{r} , interpreted as the mean return, and the covariance matrix, known as σ , finds the feasible portfolio with minimum variance that has the mean value \bar{r} . This is all subject to the weights adding to one.

Method

Xpress IVE was the best software to input the optimisation problem and to calculate the efficient frontier, as the model can be adapted in various ways. To make the model run as smoothly as possible, a loop was added that continuously calculates the revenue from a set value. The loop goes up in steps of 0.01, from zero, up until it reaches a revenue of 2.9, where the model terminates and outputs the minimum variance. These values being chosen as the average close ranges from 0 to 2.9, so the revenue would not be expected to be any higher. Inputting the covariance matrix was simplified by adding a simple command to the model. This allowed a named cell from Excel to be inputted into the model. Running the model outputted a series of figures that showed the weighting of each revenue with a joint risk.

Problems

After calculating the covariance matrix, an obvious outlier was found in the company PMO.L (Premier Oil Plc) as it was extremely higher than the rest of the companies due to the large average close value found from the historical prices as in September 2003 there was an increase of 10854.6003%. Removing this value made the efficient frontier far smoother and easier to predict a more accurate result for the minimum variance value, with maximum revenue.

There were also problems when it came to the versions of Xpress IVE that were used and inputting the covariance matrix Excel no longer worked. To solve this problem the model was reverted back to inputting the matrix manually.

Results

Running the model created an efficient frontier, shown in figure 1, that contains the calculated optimum portfolios. The results show that a return of 0.84% a month, with a minimum variance of 8.19284, is the point on the efficient frontier where the portfolio reaches its minimum -variance point, this point being where the return is at its greatest whilst giving minimum risk. This minimum-variance point contains the portfolio that is more suited to the risk averse, the risk averse aiming for a high return with as little risk as possible. Table 1 shows that calculated weightings.

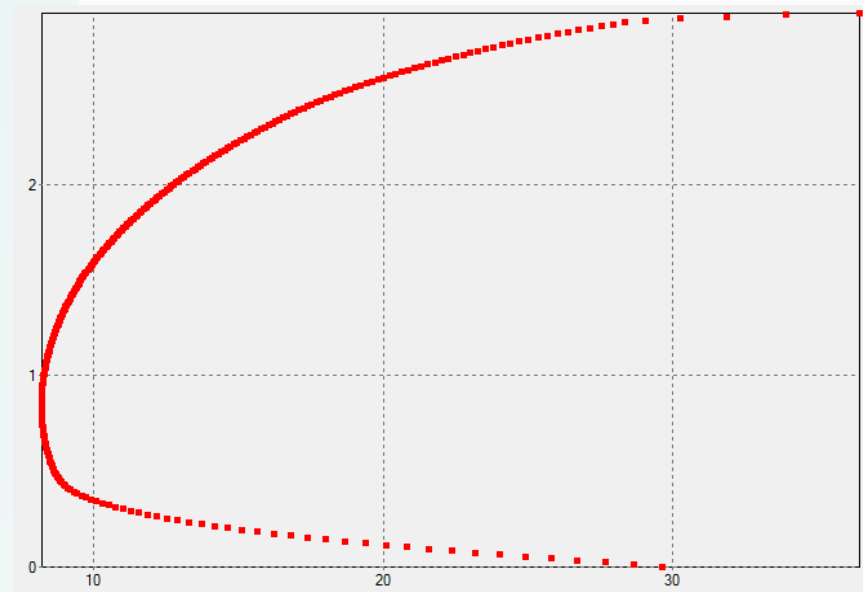


Figure 1 - Efficient Frontier of Large Scale Model

Table 1 – Weightings of the minimum-variance portfolio

SNE	UEIC	DAKT	AAPL	HAR	BAC	BARC.L	HSBA.L	LLOY.L	SAN
3.10%	0.26%	1.46%	0%	0%	1.42%	0%	21.03%	0%	0%
SNK	GRG.L	MRW.L	TSCO.L	SBRY.L	CNE.L	BP	BG.L	SIA.L	
1.96%	7.57%	25.71%	7.84%	9.31%	0%	7.85%	7.11%	5.37%	

On the other had, for the risk seeking, who are willing to take more risk in order to achieve a greater return, there are a lot more options to take. These options lie higher on the curve than for the risk averse. To get a revenue of 1.5% a month, which is a significant increase of return, the risk involved would be 9.57%, as seen below. This level of risk does not differ greatly from the 8.19% found in the optimum portfolio.

Conclusion

The aim of this project was to create a model on the Markowitz Model to calculate the optimum portfolio with minimum risk at a set value of return. This resulted in finding that the minimum-variance point had a maximum revenue of 0.84% a month that had a joint risk of 8.19284. Having a smooth efficient frontier curve has shown to provide the best outcomes as the results that are connected to these smooth curves show that these weightings are the best ways to invest into the companies. Further work that could be looked are the effects that short selling may have and how the efficient frontier depends on short selling and borrowing. Also, testing how well the whole model works in real life trading situations could be something else to look at.

References

Luenberger, D., 1995. *Investment Science*. New York: Oxford University Press.
Markowitz, H., 1952. The Journal of Finance. Volume 7, pp. 77-91.