Accretion of Planetesimals in a Gaseous Ring

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The Nebular Theory

Scientific speculation regarding the Solar System's formation is thought to have begun during the 16th century; about 200 years after the well-known astronomer, Nicholaus Copernicus, discovered that the Earth revolves around the Sun. Fast-forward to the 18th century, and the most widely accepted theory of the Solar System's formation was proposed; The Nebular Theory. Famous mathematician, Pierre-Simon de Laplace believed that the solar system first began as a nebula cloud, which is an interstellar cloud of dust, hydrogen, helium and other gasses, and from these gasses formed the sun, moons, planets and asteroids. This was a major turning point in man's discovery into the formation of the solar system.

A Modern Laplacian Theory

In 1980, mathematician, Dr Andrew Prentice, attempted to construct a modern interpretation of Laplace's Nebular Theory. In the Modern Laplacian Theory, after the collapse of an interstellar cloud, it leaves behind a series of gaseous rings. This study means to examine the collapse process, and to investigate how a single particle of rock or ice, which is situated in the plane of a gaseous ring, will eventually join an existing Embryo whilst being under the influence of the gravitational pull of the Sun and the existing Embryo as well as undergoing gas drag of the surrounding gas.

The Collapse Process

For a cloud with a particular radius and temperature, there is a critical mass known as Jean's Mass. If a cloud exceeds this mass, it will become unstable and collapse under gravitational pull.

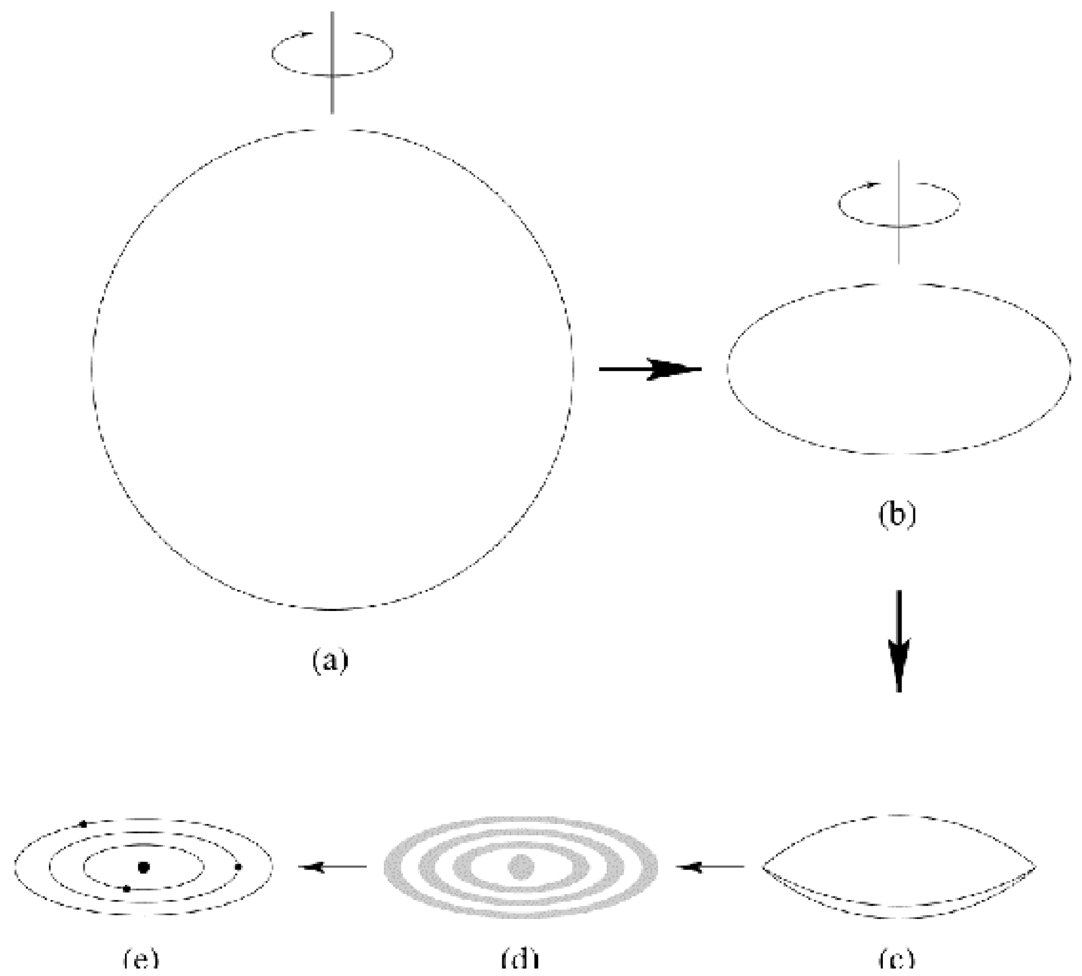

Figure 1: A schematic representation of the Laplace Nebular Theory. (a) A slowly rotating and collapsing cloud of gas and dust, (b) The collapsing nebula flattens along its rotation axis to form an oblate spheroid, (c) Formation of a lenticular shape, (d) A series of rings is left behind by the contracting core, (e) one planet is formed from the material in each ring.

Aims and Objectives

- To carry out a thorough literature review
- Read and fully understand the paper written by Dr Andrew Prentice entitled: *Accretion of Planetesimals in a Gaseous Ring*
- Obtain analytical solutions to the equations of motion, found as 10a & 10b in Prentice's paper, under the conditions that the gas drag is much greater than 1, and then the gas drag is much less than 1
- To obtain solutions to equations 21, 24, 35, 36

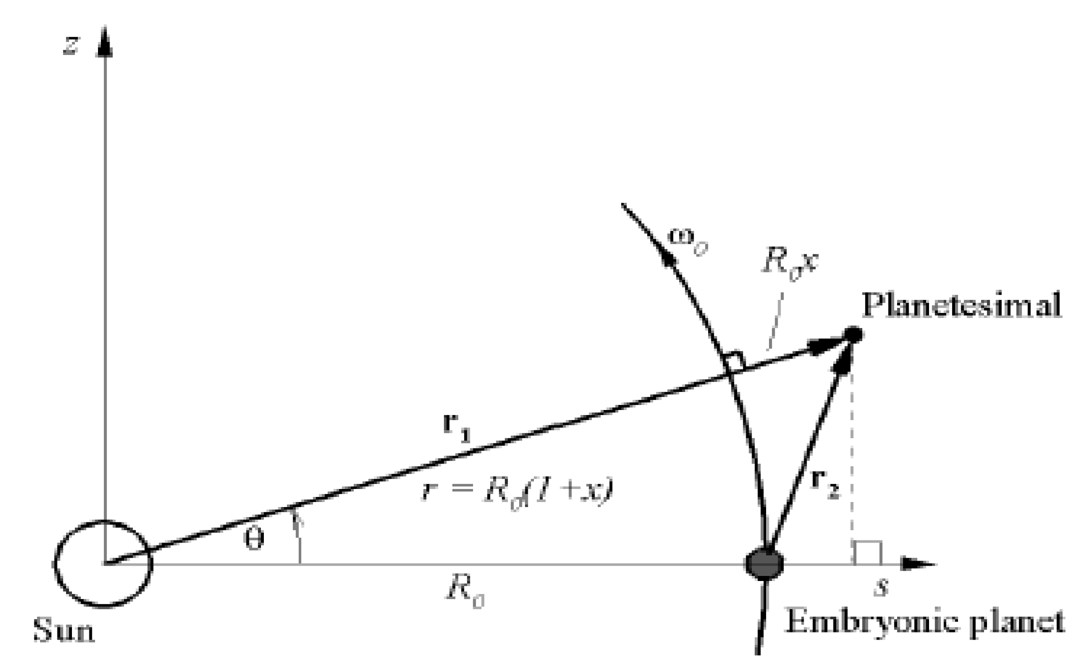

Figure 2: This is a diagram showing the local polar coordinate system *(r,ϑ)* of the Embryo and Planetesimal as shown in the Sun's frame of reference. It should be stated that *ϑ* is the angular coordinate and *r* is the radial coordinate and *z* denotes the usual y coordinate.

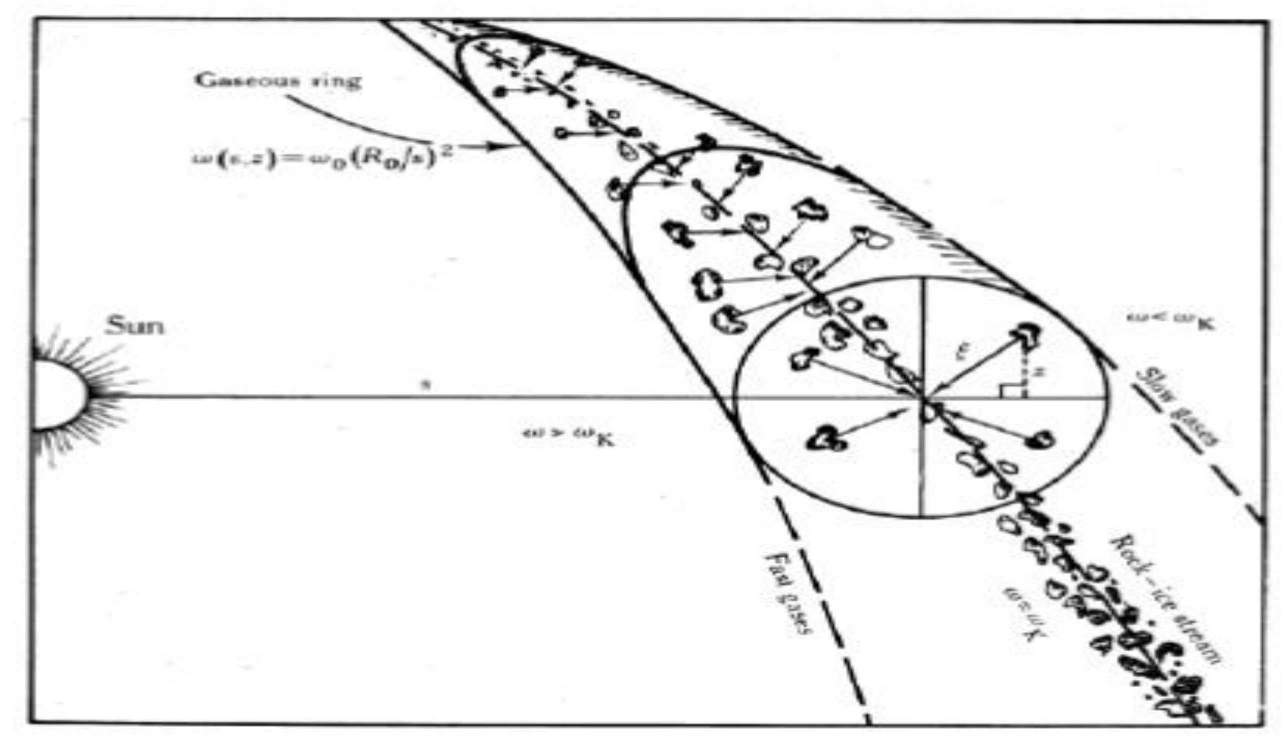

Figure 3: Schematic illustration of the gravitational settling of condensed particles onto the mean Keplerian orbit of the gaseous ring to form a concentrated orbiting stream of planetesimals.

Equations of Motion

$$\ddot{x} - (1+x)(\omega_0 + \dot{\theta})^2 = -{\omega_0}^2/(1+x)^2 - \varepsilon{\omega_0}^2\left(x - 2sin^2\frac{\theta}{2}\right)/d^3 - k\dot{x}\sqrt{\dot{x}^2 + \dot{u}^2} \quad [1]$$

$$(1+x)\ddot{\theta} + 2\dot{x}\left(\omega_0 + \dot{\theta}\right) = -\varepsilon{\omega_0}^2 sin\,\theta/d^3 - ku\sqrt{\dot{x}^2 + u^2} \quad [2]$$

The equations of motion can be linearized to discover the planetesimal's Azimuthal and Trajectory solutions.

Results

The equations of motion were examined under two different cases: $k\theta_0 \ll 1$ and $k\theta_0 \gg 1$. The solutions can be found as the following:

Azimuthal Solution:

Case (i), $k\theta_0 \ll 1$:

$$\omega_0 t = \left(\frac{{\theta_0}^3}{2\varepsilon}\right)^{1/2}\left[\frac{\pi}{2} - sin^{-1}\sqrt{\theta/\theta_0} + \{(\theta/\theta_0)(1-\theta/\theta_0)\}^{1/2}\right]$$

Case (ii), $k\theta_0 \gg 1$

$$\theta(t) = \theta_0\left[1 - \left(2\omega_0/{\theta_0}^2\right)(\varepsilon/k)^{1/2}t\right]^{1/2}$$

Trajectory Solution:

Case (i), $k\theta_0 \ll 1$

$$x(\theta) = -\left(\frac{\theta_0}{2\varepsilon}\right)^{1/2}\theta\left[2\left(\frac{\theta_0}{\theta} - 1\right)^{1/2} + sin^{-1}\left(\frac{\theta}{\theta_0}\right) - \frac{\pi}{2} - \left[\frac{\theta}{\theta_0} - \left(1 - \frac{\theta}{\theta_0}\right)\right]^{1/2}\right]$$

Case (ii), $k\theta_0 \gg 1$

$$x(\theta) = 2\theta(\theta_0 - \theta)(\varepsilon k)^{-\frac{1}{2}}$$

Conclusion

For the azimuthal solution, the equations can be solved by computing them into a mathematical program. However, to determine whether the Planetesimal is either eventually captured or avoids being caught by the Embryo, needs a detailed numerical integration of the complete nonlinear equations of motion.

Prentice discovered that if the condition ${\theta_0}^3 \ll \frac{1}{3}\varepsilon$ is met and that the gas drag is much greater than 1 (where ε is the mass of the Embryo divided by the mass of the Sun), then the Planetesimal is directly accreted by the Embryo, given that $\theta \to 0$. It is clear that having the Planetesimal experience a high gas drag is crucial for its accretion.

When the gas friction is higher, it is harder for the Planetesimal to escape the mean circular orbit of the Embryo. Therefore when the Planetesimal is located inside the Embryo's gravitational sphere of influence, the surrounding gaseous drag manages to secure its accretion, hence resisting the Coriolis force.

Assumptions and Limitations

- Model only acknowledges the gravitational pull of one Embryonic Planet and the Sun
- Results are only approximations due to the fact that the equations of motion were linearized and so are only true in narrow regimes of the various parameters
- This project does not take turbulent stress into account; According to Dr Andrew Prentice, there existed extremely powerful convective currents within the primitive solar cloud, and these supersonic currents create turbulent stress which provide the mechanism for shedding individual gas rings

Further reading

Bierbrauer, F. (2015) Introductory Notes. Manchester Metropolitan Univeristy. 15- page handout, distributed in May 2015 [Online] [Accessed in January 2016.

Prentice, A.J.R. Accretion of Planetesimals in a Gaseous Ring, Aust. J. Phys., 33 (1980), 623-637.

Woolfson, M.M. (2000) The Origin and Evolution of the Solar System. London: Institute of Physics Publishing