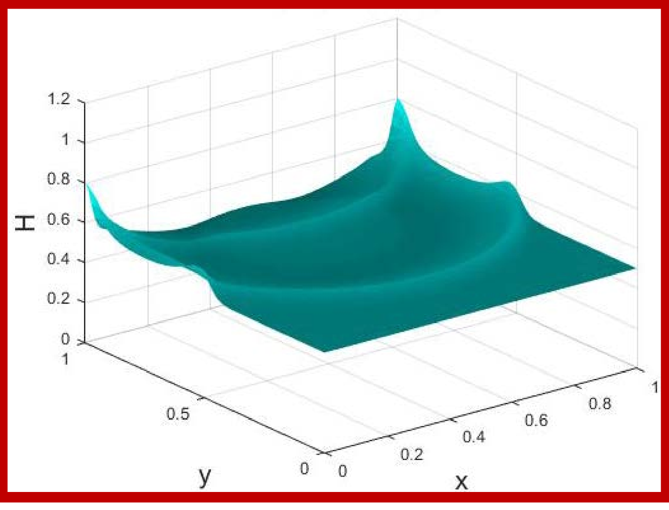

The Shallow Water Equations

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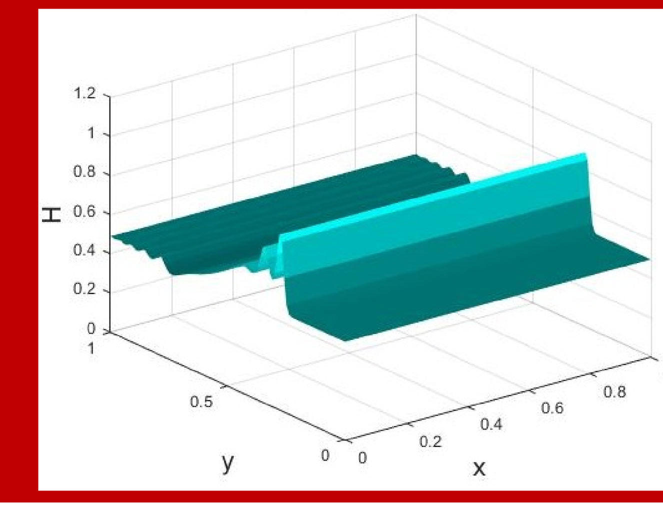

Introduction

By creating a simulation of water, a prediction can be made as to how a water wave forms and travels. Such a model can be used to forecast real-world events including tides in an ocean, the form of a devastating tsunami and even the event of a water dam breaking (Toro, 2011).

A system of hyperbolic partial differential equations (PDEs), named the "*shallow water equations*" (SWEs), describe the motion of water in shallow environments. The term *shallow* applies to water that has an extremely low height-to-width ratio.

Another example of a PDE that can be used to create a simulation of water is the *advection equation*. Although such a model can be seen as a primitive and unrealistic, the advection equation produces a simulation of a wave travelling across a domain at a constant speed.

Approach

The SWEs have two variations in the form of 1D and 2D. Such dimensions describe the surface of the wave and not the space, as one might first think.

By using a finite difference method, with the incorporation of a finite difference scheme (FDS), approximate solutions to the 1D and 2D variation of the SWEs and the advection equation are found. The constructed solution is therefore use to create a simulation of water.

Water Model Equations

1D advection: $U_t + vU_x = 0.$ (Fitzpatrick, 2006)

1D SWEs:

$$H_t + (vH)_x = 0,$$
$$(vH)_t + (v^2 H + \frac{1}{2} gH^2)_x = gHh_x.$$

(Boer,2003)

2D SWEs:

$$H_t + (vH)_x + (wH)_y = 0,$$
$$(vH)_t + (v^2 H + \frac{1}{2} gH^2)_x + (vwH)_y = gHh_x$$
$$(wH)_t + (vwH)_x + (w^2 H + \frac{1}{2} gH^2)_y = gHh_y.$$

(Toro,2011)

Solutions

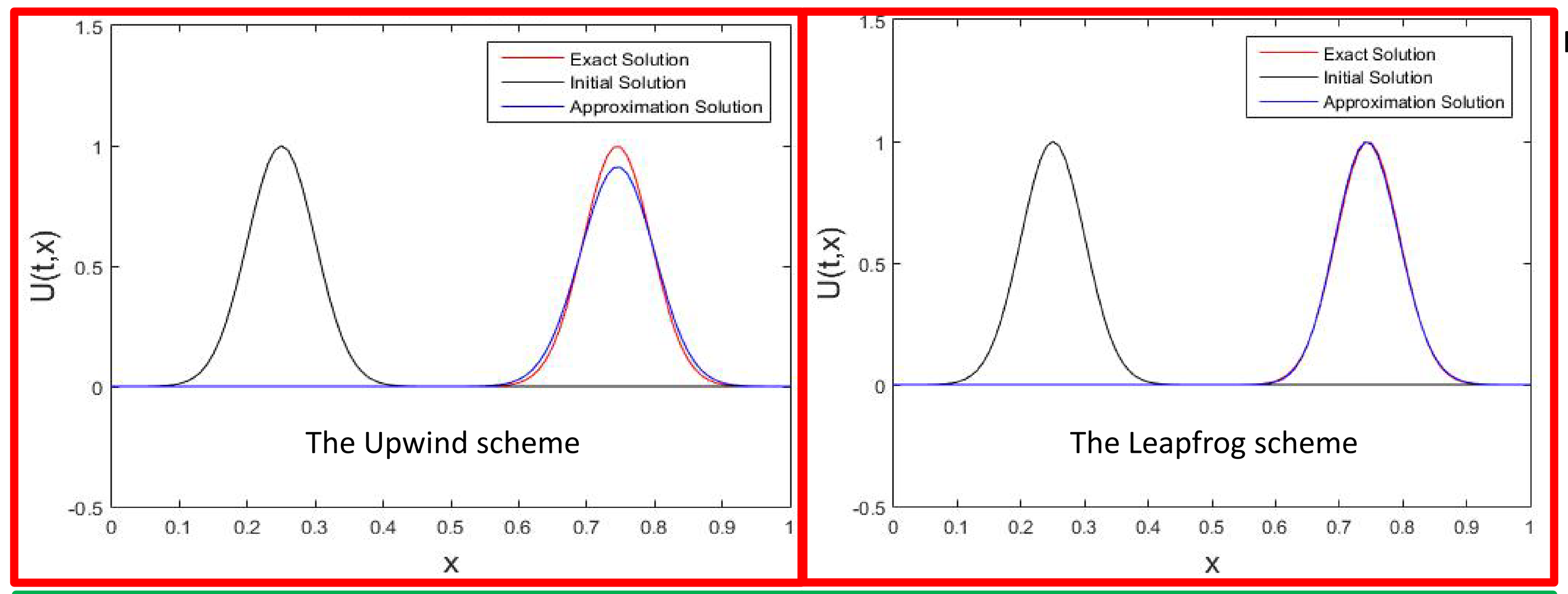

Fig.1

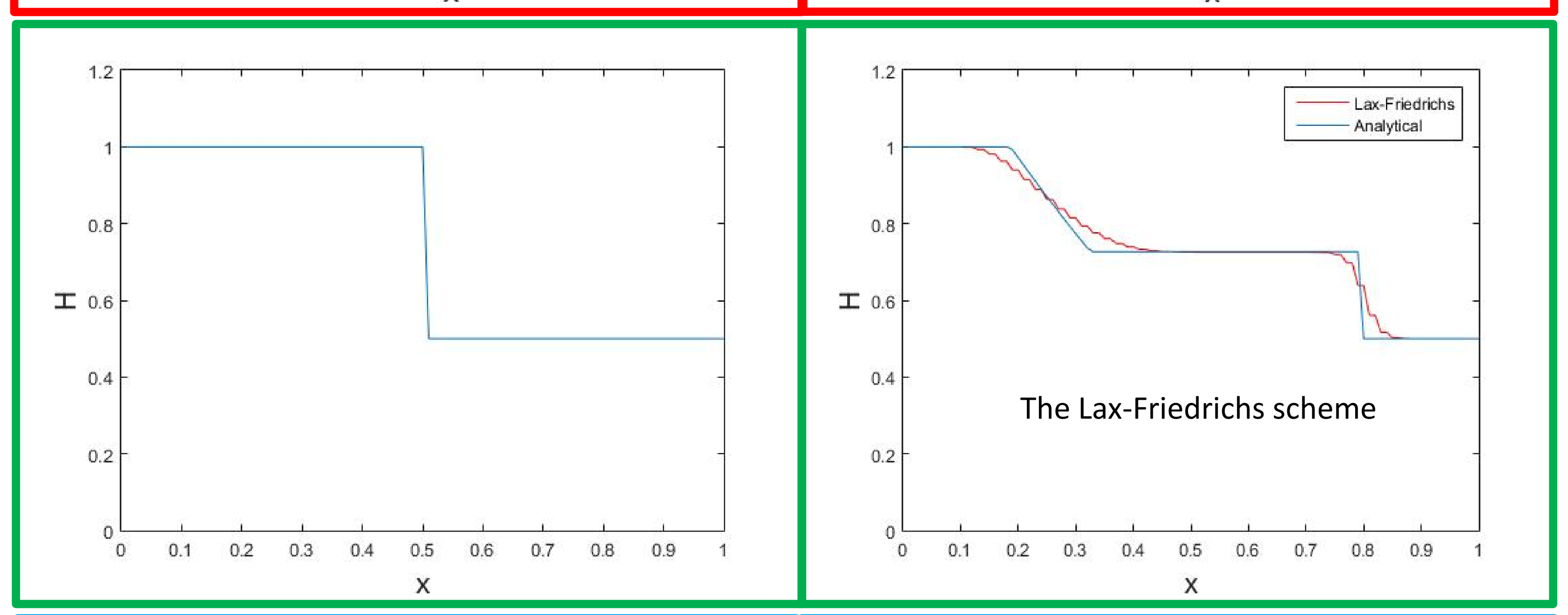

Fig.2

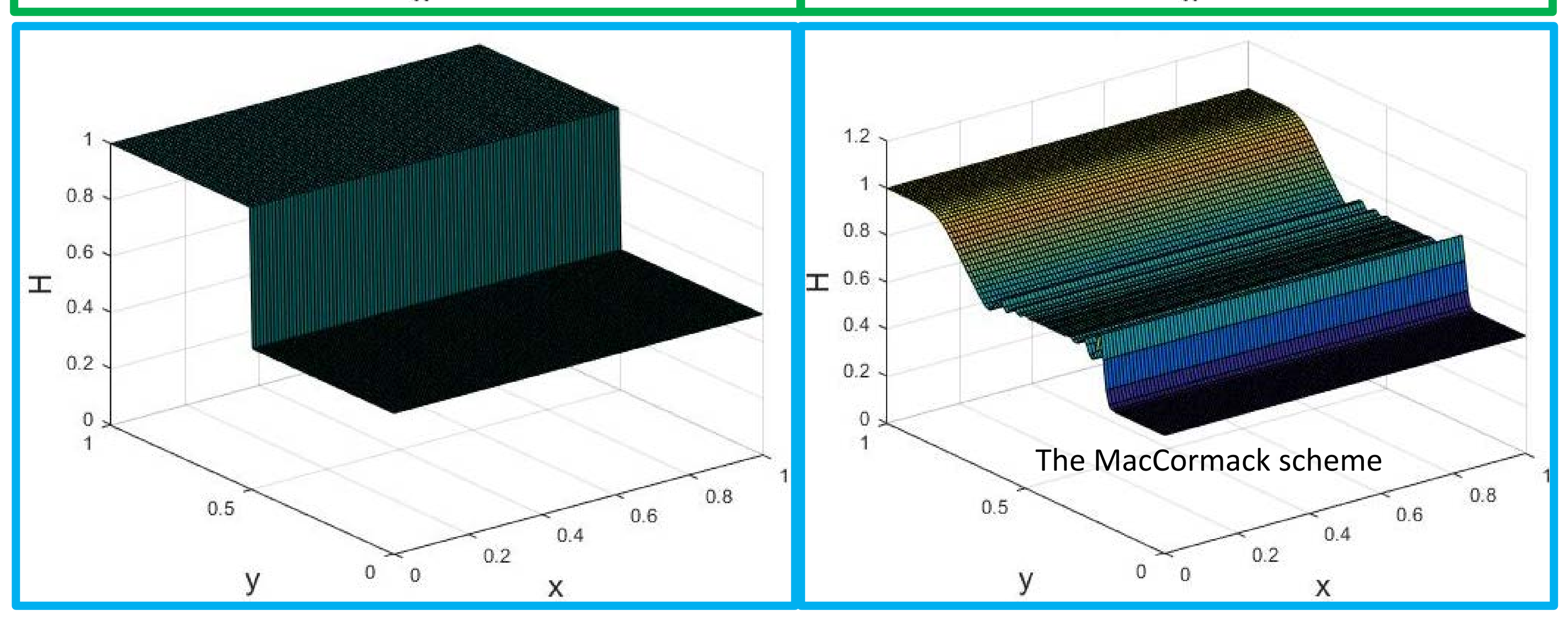

Fig.3

Conclusions

This project has shown that multiple models of water can be created through approximating a solution to a PDE. Simulations were produced that show a wave propagating across a domain, a 1D dam breaking, a 2D dam breaking, a Gaussian curve collapsing and a primitive simulation of a tsunami wave travelling across a domain. Example solutions can be found Figures 1,2,3.

It was also found that second-order accurate FDS's produced the best results. Occurrences of dissipation were found commonly in the first-order accurate schemes, therefore producing undesirable simulations.

Problems / Limitations

To increases the accuracy of results, certain error removing techniques can be used to remove unwanted oscillations around abrupt changes in gradient.

Unfortunately, I was unable to implement such methods due to limiting time constraints but could possibly be re-visited in the future.

References

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